

APPENDIX B

PRACTICE PLUS

$$(43) \begin{cases} w+x+y+z=4 \\ 2w+x-2y-z=0 \\ w-2x-y-2z=-2 \\ 3w+2x+y+3z=4 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 2 & 1 & -2 & -1 & 0 \\ 1 & -2 & -1 & -2 & -2 \\ 3 & 2 & 1 & 3 & 4 \end{array} \right] \xrightarrow{\substack{-2R_1+R_2 \\ -1R_1+R_3 \\ -3R_1+R_4}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & -1 & -4 & -3 & -8 \\ 0 & -3 & -2 & -3 & -6 \\ 0 & -1 & -2 & 0 & -8 \end{array} \right] \xrightarrow{-1R_2}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4 & 3 & 8 \\ 0 & -3 & -2 & -3 & -6 \\ 0 & -1 & -2 & 0 & -8 \end{array} \right] \xrightarrow{\substack{3R_2+R_3 \\ 1R_2+R_4}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4 & 3 & 8 \\ 0 & 0 & 10 & 6 & 18 \\ 0 & 0 & 2 & 3 & 0 \end{array} \right] \xrightarrow{\frac{1}{10}R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4 & 3 & 8 \\ 0 & 0 & 1 & \frac{3}{5} & \frac{9}{5} \\ 0 & 0 & 2 & 3 & 0 \end{array} \right] \xrightarrow{-2R_3+R_4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4 & 3 & 8 \\ 0 & 0 & 1 & \frac{3}{5} & \frac{9}{5} \\ 0 & 0 & 0 & \frac{9}{5} & -\frac{18}{5} \end{array} \right] \xrightarrow{\frac{5}{9}R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 0 & 1 & 4 & 3 & 8 \\ 0 & 0 & 1 & \frac{3}{5} & \frac{9}{5} \\ 0 & 0 & 0 & 1 & -2 \end{array} \right]$$

$$\boxed{z = -2}$$

$$\begin{aligned} y + \frac{3}{5}z &= \frac{9}{5} \\ y + \frac{3}{5}(-2) &= \frac{9}{5} \\ y - \frac{6}{5} &= \frac{9}{5} \\ y &= \frac{15}{5} = 3 \\ \boxed{y = 3} \end{aligned}$$

SOLUTION

$$(1, 2, 3, -2)$$

$$\begin{aligned} w+x+y+z &= 4 \\ w+2+3+(-2) &= 4 \\ w+3 &= 4 \\ \boxed{w = 1} \end{aligned}$$

Solution Set $\rightarrow \{(1, 2, 3, -2)\}$

$$\begin{aligned} x+4y+3z &= 8 \\ x+4(3)+3(-2) &= 8 \\ x+12-6 &= 8 \\ x+6 &= 8 \\ x &= 8-6 \\ \boxed{x = 2} \end{aligned}$$

Appendix B Practice PLUS

44

$$\begin{cases} w+x+y+z=5 \\ w+2x-y-2z=-1 \\ w-3x-3y-z=-1 \\ 2w-x+2y-z=-2 \end{cases}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 1 & 2 & -1 & -2 & -1 \\ 1 & -3 & -3 & -1 & -1 \\ 2 & -1 & 2 & -1 & -2 \end{array} \right] \xrightarrow{\substack{-1R_1+R_2 \\ -1R_1+R_3 \\ -2R_1+R_4}} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & -4 & -4 & -2 & -6 \\ 0 & -3 & 0 & -3 & -12 \end{array} \right] \xrightarrow{\substack{4R_2+R_3 \\ 3R_2+R_4}}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & -12 & -14 & -30 \\ 0 & 0 & -6 & -12 & -30 \end{array} \right] \xrightarrow{-\frac{1}{12}R_3} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & 1 & \frac{7}{6} & \frac{5}{2} \\ 0 & 0 & -6 & -12 & -30 \end{array} \right] \xrightarrow{6R_3+R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & 1 & \frac{7}{6} & \frac{5}{2} \\ 0 & 0 & 0 & -5 & -15 \end{array} \right] \xrightarrow{-\frac{1}{5}R_4} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 5 \\ 0 & 1 & -2 & -3 & -6 \\ 0 & 0 & 1 & \frac{7}{6} & \frac{5}{2} \\ 0 & 0 & 0 & 1 & 3 \end{array} \right]$$

$\boxed{z=3}$ ✓

$$\begin{aligned} y + \frac{7}{6}z &= \frac{5}{2} \\ y + \frac{7}{6}(3) &= \frac{5}{2} \\ y + \frac{7}{2} &= \frac{5}{2} \\ y &= \frac{5}{2} - \frac{7}{2} \\ y &= \frac{5-7}{2} = \frac{-2}{2} = -1 \end{aligned}$$

$\boxed{y=-1}$ ✓

$$\begin{aligned} x - 2y - 3z &= -6 \\ x - 2(-1) - 3(3) &= -6 \\ x + 2 - 9 &= -6 \\ x - 7 &= -6 \\ x &= -6 + 7 \\ \boxed{x=1} \end{aligned}$$

$$\begin{aligned} w+x+y+z &= 5 \\ w+1+(-1)+3 &= 5 \\ w+3 &= 5 \\ w &= 5-3=2 \\ \boxed{w=2} \end{aligned}$$

Solution $\rightarrow (2, 1, -1, 3)$

Solution set $\rightarrow \{(2, 1, -1, 3)\}$